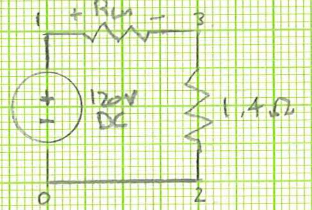
VOLTAGE DIVIDER

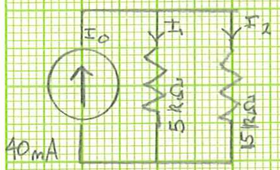


$V_o = 120V$

$V_{23} = 100V$

$V_i = V_o \frac{R_i}{R_{eq}}$ $\qquad 100 = 120 \cdot \frac{1.4}{1.4 + R_{Ln}} \longrightarrow R_{Ln} = 0.28\,\Omega$

CURRENT DIVIDER

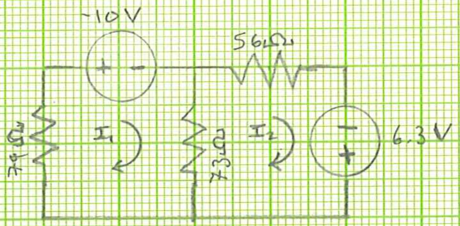


$I_i = I_o \frac{R_{eq}}{R_i}$

$I_1 = 40mA \frac{3750}{5000} = 30mA$

$I_2 = 40mA \frac{3750}{15000} = 10mA$

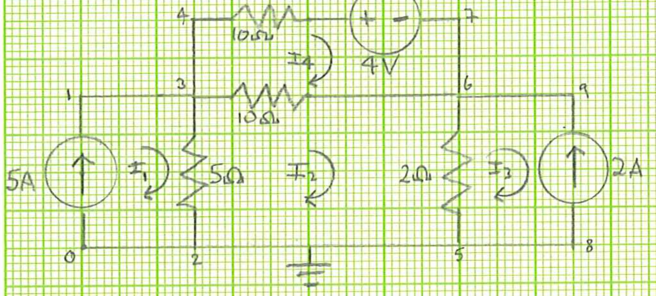
MESH CURRENT



$I_1: 79\Omega I_1 + -10V + 73\Omega (I_1 - I_2) = 0$

$I_2: 73\Omega (I_1 - I_2) + 56\Omega I_2 - 6.3V = 0$

$rref \begin{bmatrix} 79+73 & -73 & 10 \\ -73 & 73+56 & 6.3 \end{bmatrix} \longrightarrow \begin{array}{l} I_1 = 0.123\,A \\ I_2 = 0.118\,A \end{array}$



$I_1: -V_{01} + 5\Omega (I_1 - I_2) = 0$

$I_2: 5\Omega (I_2 - I_1) + 10\Omega (I_2 - I_4) + 2\Omega (I_2 - I_3) = 0$

$-5\Omega I_1 + 17\Omega I_2 - 2\Omega I_3 - 10\Omega I_4 = 0$

$I_3: 2\Omega (I_3 - I_2) + V_{98} = 0$

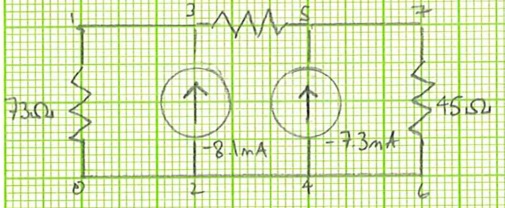
$I_4: 10\Omega I_4 + 4V + 10\Omega (I_4 - I_2) = 0$

$-10\Omega I_2 + 20\Omega I_4 = -4V$

$I_1 = 5A \quad I_3 = -2A \quad I_4 = \frac{-4 + 10 I_2}{20} = 0.59A$

$I_2 = 1.58\,A$

NODE VOLTAGE



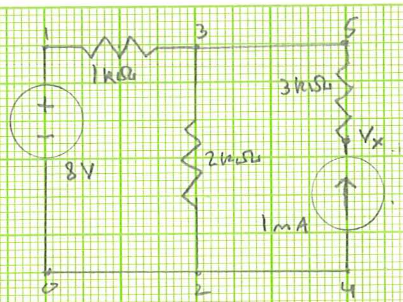
$V_3: I_{3\uparrow} = -8.1mA \quad I_{3\leftarrow} = \frac{V_3 - 0}{73\Omega} \quad I_{3\rightarrow} = \frac{V_3 - V_5}{99\Omega} \qquad \sum_i I_i = 0$

$V_5: I_{5\uparrow} = -7.3mA \quad I_{5\leftarrow} = \frac{V_5 - V_3}{99\Omega} \quad I_{5\rightarrow} = \frac{V_5 - 0}{45\Omega} \qquad \sum_i I_i = 0$

$V_3 \left(\frac{1}{73} + \frac{1}{99}\right) + V_5 \left(-\frac{1}{99}\right) = 8.1mA$

$V_3 \left(-\frac{1}{99}\right) + V_5 \left(\frac{1}{99} + \frac{1}{45}\right) = 7.3mA$

$rref \begin{bmatrix} \frac{1}{73} + \frac{1}{99} & -\frac{1}{99} & 8.1 \times 10^{-3} \\ -\frac{1}{99} & \frac{1}{99} + \frac{1}{45} & 7.3 \times 10^{-3} \end{bmatrix} \longrightarrow \begin{array}{l} V_3 = 0.503\,V \\ V_5 = 0.383\,V \end{array}$



NODE VOLTAGE

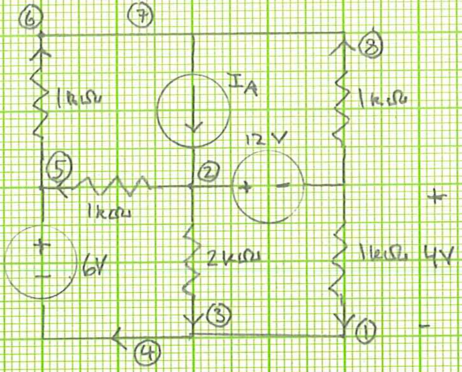
$V_3: I_{3\leftarrow} = \frac{V_3 - 8}{1000}$ $I_{3\downarrow} = \frac{V_3 - 0}{2000}$ $I_{3\rightarrow} = \frac{V_3 - V_x}{3000}$ $\Sigma I_3 = 0$

$V_x: I_{\uparrow} = \frac{V_x - V_3}{3000}$ $I_{x\downarrow} = -1\times10^{-3}$ $\Sigma I_x = 0$

$$V_3\left(\frac{1}{1000} + \frac{1}{2000} + \frac{1}{3000}\right) + V_x\left(-\frac{1}{3000}\right) = \frac{8}{1000}$$

$$V_3\left(-\frac{1}{3000}\right) + V_x\left(\frac{1}{3000}\right) = 1\times10^{-3}$$

$$\mathrm{rref}\begin{bmatrix} \frac{1}{1000}+\frac{1}{2000}+\frac{1}{3000} & -\frac{1}{3000} & \frac{8}{1000} \\ -\frac{1}{3000} & \frac{1}{3000} & 1\times10^{-3} \end{bmatrix} \rightarrow \begin{matrix} V_3 = 6V \\ V_x = 9V \end{matrix}$$

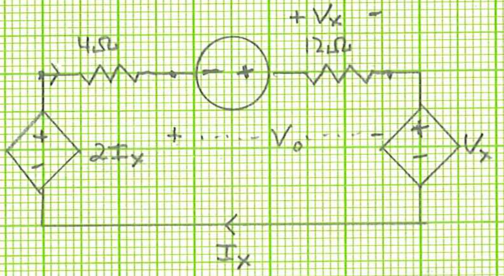


CIRCUIT ANALYSIS

1. $\frac{4V}{1000\Omega} = 4mA$
2. $4V + 12V = 16V$
3. $\frac{16V}{2000\Omega} = 8mA$
4. $4mA + 8mA = 12mA$
5. $\frac{16V - 6V}{1000\Omega} = 10mA$
6. $12mA + 10mA = 22mA$
7. $6V - 22mA \cdot 1000\Omega = -16V$
8. $\frac{4V - -16V}{1000\Omega} = 20mA$

$I_A = 22mA + 20mA = 42mA$

DEPENDENT SOURCE

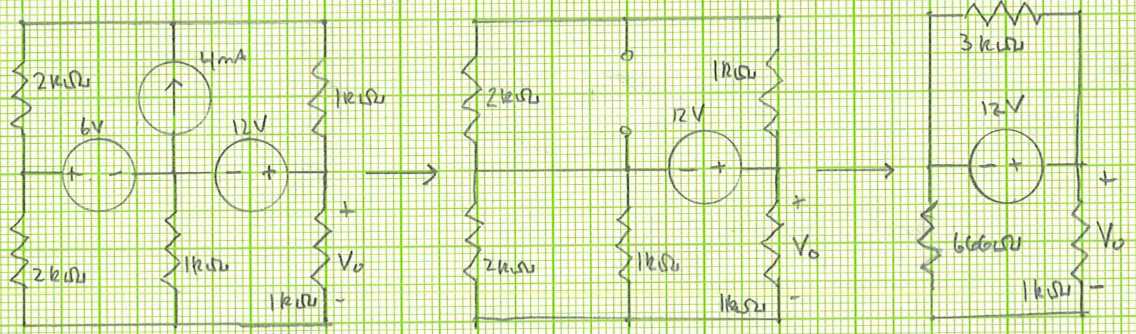


$-2I_x + 4I_x - 12 - V_x + V_x = 0 \rightarrow I_x = 6$

$V_x = 6A \cdot 12\Omega = 72V$

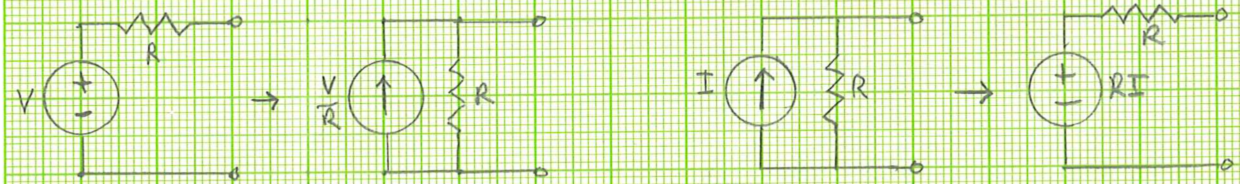
$V_o = -12 - {}^-72 = 60V$

SUPERPOSITION

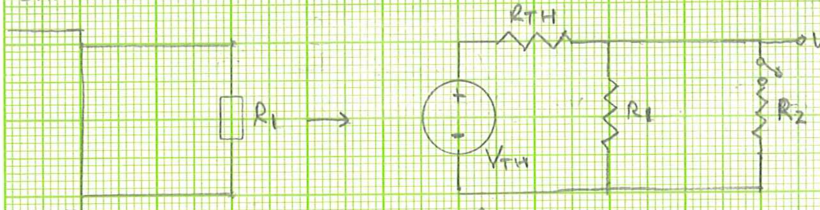


FIND PORTION OF V_o FROM 12V SOURCE: $V_o = \frac{1000}{1000+666}\Omega\ 12V = 7.2V$

SOURCE TRANSFORMATION



Ex. 1



$R_1 = 20\,k\Omega$
$V_2 = 25\,V$

$R_2 = 2\,k\Omega$
$V_{2L} = 10\ V$ (R_2 ON)

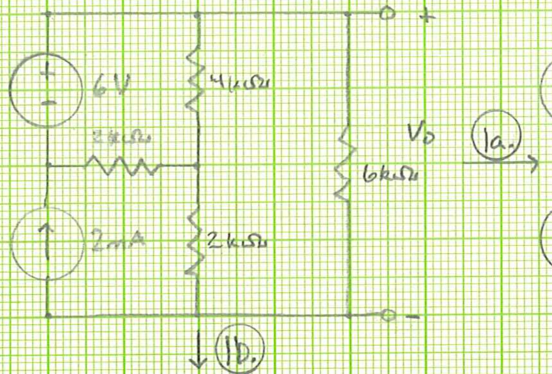
THÉVENIN

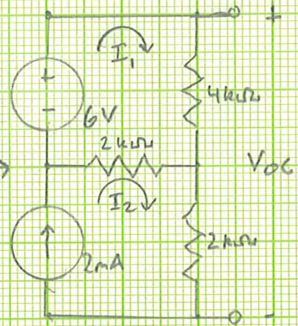
LET $d = \left[R_1 \cdot V_{2L} + R_2 (V_{2L} - V_2)\right]^{-1}$

$V_{TH} = V_2 \cdot V_{2L} \cdot R_1 \cdot d = 29.4\,V$

$R_{TH} = R_1 \cdot R_2 \cdot (V_2 - V_{2L}) \cdot d = 3529\,\Omega$

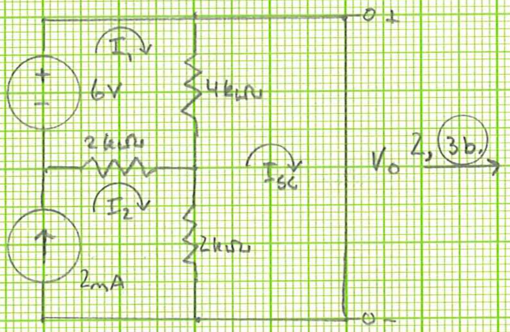
Ex. 2

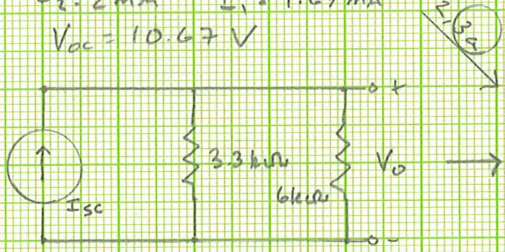


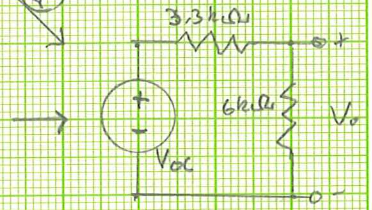


$0 = -6 + I_1(4000) + (I_1 - I_2)(2000)$
$I_2 = 2\,mA \qquad I_1 = 1.67\,mA$
$V_{OC} = 10.67\,V$

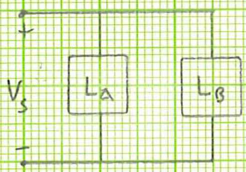
$R_{TH} = 3333.3\,\Omega$



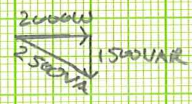




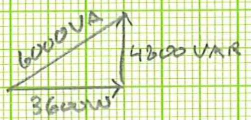
$0 = -6 + (I_1 - I_{SC})(4000) + (I_1 - I_2)(2000)$
$0 = -(I_1 - I_{SC})(4000) - (I_2 - I_{SC})(2000)$
$I_1 = 3.8\,mA \qquad I_{SC} = 3.2\,mA$



A.

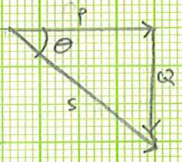


B.



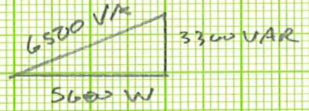
POWER

LEADING

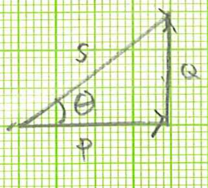


LOAD A: 2kW, p.f. = 0.8 LEADING $|S| = \frac{P}{p.f.} \rightarrow S = 2500VA$
LOAD B: 6kVA, p.f. = 0.6 LAGGING $P = |S| p.f. \rightarrow P = 3600V$

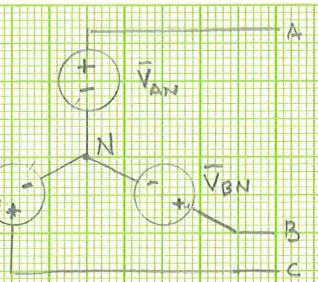
S_T: DON'T SUM S_i; ADD VECTORS



LAGGING



3φ POWER

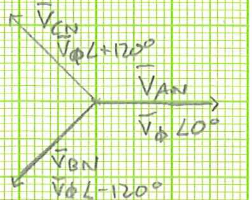


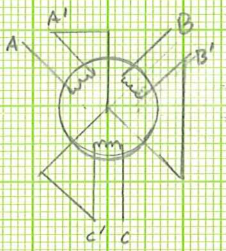
$\bar V_{AB} = \bar V_{AN} - \bar V_{BN} = \bar V_{AN}\sqrt{3}\angle 30^\circ$

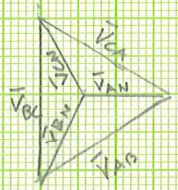
$\bar V_{BC} = \bar V_{BN} - \bar V_{CN} = \bar V_{BN}\cdot\sqrt{3}\angle 30^\circ$

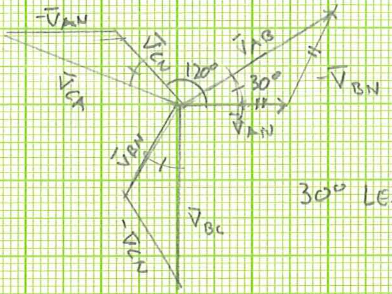
$\bar V_{CA} = \bar V_{CN} - \bar V_{AN} = \bar V_{CN}\cdot\sqrt{3}\angle 30^\circ$

$\bar V_{AN} + \bar V_{BN} + \bar V_{CN} = 0 \quad \bar V_L = \bar V_\phi\cdot\sqrt{3}\angle 30^\circ$



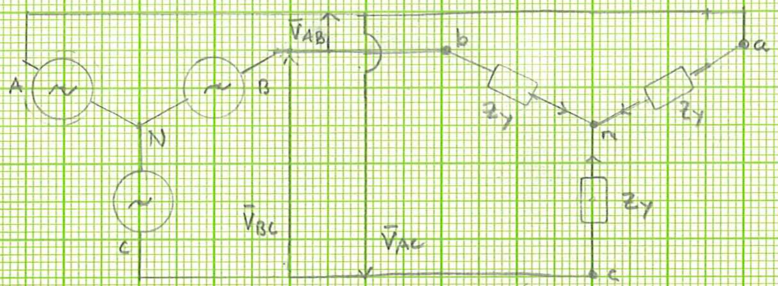






30° LEADING

Y - Y

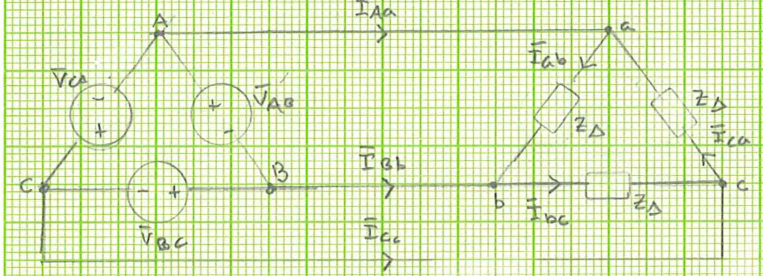


$\bar I_{an} = \frac{\bar V_{AN}}{Z_Y} \quad \bar I_{bn} = \frac{\bar V_{BN}}{Z_Y} \quad \bar I_{cn} = \frac{\bar V_{CN}}{Z_Y} \quad \Delta\phi = 120^\circ$

$S_{\phi a} = \bar V_{an}\bar I_{an}^* \qquad S_T = 3S_\phi$

$\bar V_{AB} = \bar V_\phi\cdot\sqrt{3}\angle 30^\circ \qquad \bar V_\phi = \bar V_{AN}$

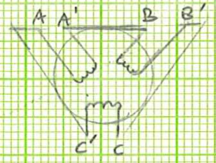
Δ - Δ

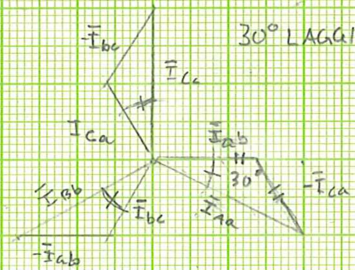


$\bar I_{ab} = \frac{\bar V_{AB}}{Z_\Delta} \quad \bar I_{bc} = \frac{\bar V_{BC}}{Z_\Delta} \quad \bar I_{ca} = \frac{\bar V_{CA}}{Z_\Delta} \quad \Delta\phi = 120^\circ$

$S_{\phi a} = \bar V_{AB}\bar I_{ab}^* \qquad S_T = 3S_\phi$

$\bar V_{ab} = \bar V_{AB}$



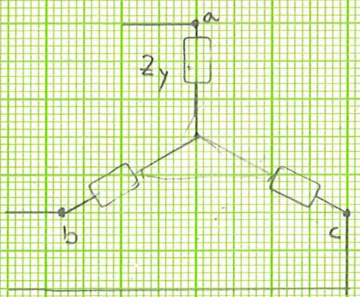


30° LAGGING

$\bar I_{Aa} = \bar I_{ab} - \bar I_{ca} = \bar I_{ab}\sqrt{3}\angle 30^\circ$

$\bar I_{Bb} = \bar I_{bc} - \bar I_{ab} = \bar I_{bc}\sqrt{3}\angle 30^\circ$

$\bar I_{Cc} = \bar I_{ca} - \bar I_{bc} = \bar I_{ca}\sqrt{3}\angle 30^\circ$



$S_{\phi a} = \bar V_{an}\bar I_{an}^*$

$S_T = 3S_\phi$

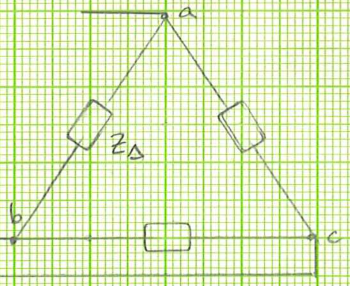
$S_T = 3|\bar V_{AN}||\bar I_{AN}|\angle\phi_Z$

$S_T = 3\left|\frac{V_L}{\sqrt{3}}\right||I_L|\angle\phi_Z$

$S_T = \sqrt{3}|V_L||I_L|\angle\phi_Z$

$I_L = \bar I_{an} = \bar I_{nb} = \bar I_{nc}$

$Z_Y = \frac{Z_\Delta}{3}$



$S_{\phi a} = \bar V_{ab}\bar I_{ab}^*$

$S_T = 3S_\phi$

$S_T = 3|\bar V_{ab}||\bar I_{ab}|\angle\phi_Z$

$S_T = 3|\bar V_L|\left|\frac{I_L}{\sqrt{3}}\right|\angle\phi_Z$

$S_T = \sqrt{3}|V_L||I_L|\angle\phi_Z$

$V_L = V_{ab} = V_{bc} = V_{ca}$

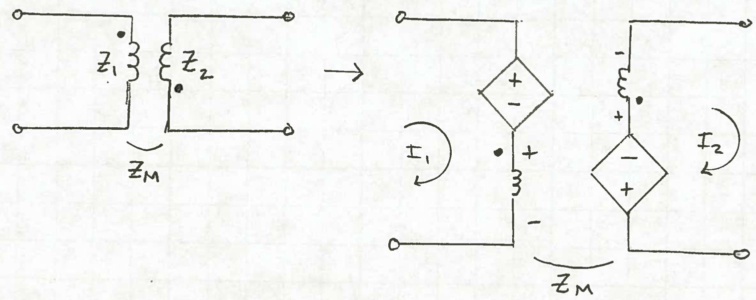
$Z_\Delta = 3Z_Y$

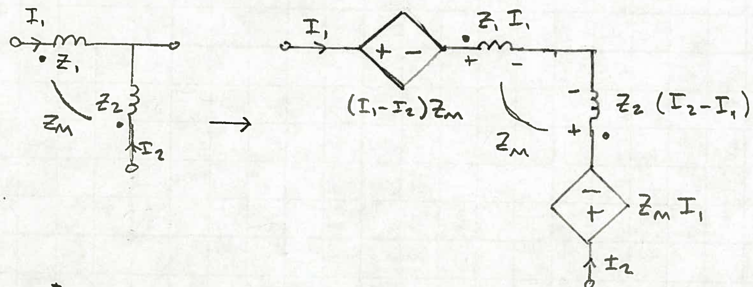
MUTUAL INDUCTANCE

CURRENT INTO DOT PRODUCES POSITIVE CURRENT OUT OF THE OTHER

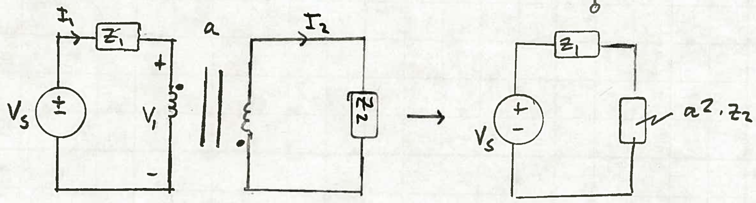
CURRENT FLOWS THROUGH INDUCTOR IN DIRECTION OF DROP

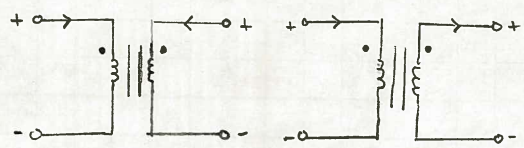
$\pm L_m I_1 I_2$ DEPENDING ON WHETHER CURRENTS IN/OUT OF DOTS HAVE SAME DIRECTION





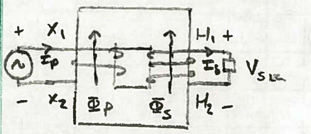
IDEAL TRANSFORMER





$N_{t1} I_1 = -N_{t2} I_2$ $\quad$ $N_{t1} I_1 = N_{t2} I_2$

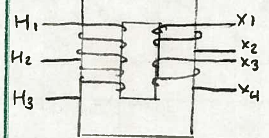
1Φ TRANSFORMER



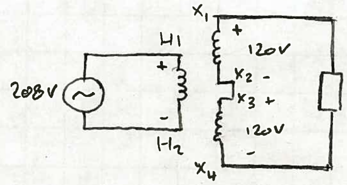
$X_1(+) \rightarrow H_1(+)$

CURRENT IN X1 → CURRENT OUT H_1

CURRENT FLOWS IN DIRECTION OF V DROP

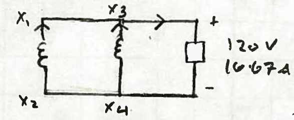


$H_1 - H_2$ 208V
$H_1 - H_2$ 104V
$X_1 - X_2$ 120V
$X_3 - X_4$ 120V
1 kVA



$H_1(+) \rightarrow X_1(+), X_3(+)$

208/240 V CONFIG.



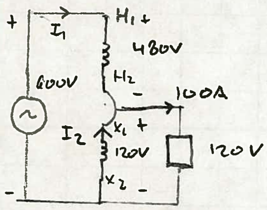
$I_{RATED} = \frac{1\,kVA}{120V} = 8.33A$ (COIL)

208/120V CONFIG.

480/120 V 60Hz 9.6kVA → 600/120V

AUTO TRANSFORMER

SERIESED COILS



I_1 RATED = $\frac{9600VA}{480V} = 20A$

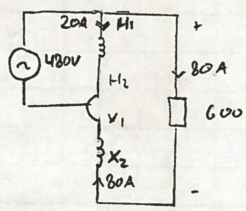
I_2 RATED = $\frac{9600VA}{120V} = 80A = 4I_1$

CURRENT IN H_1 → CURRENT OUT X_1

$S_T = 12kVA$

V RATIO IS ALWAYS THE SAME. CURRENT CANNOT BE HIGHER.

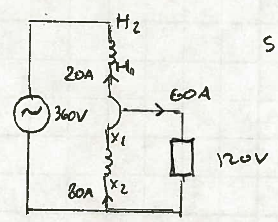
480/600V



I_{1R}, I_{2R} DON'T CHANGE

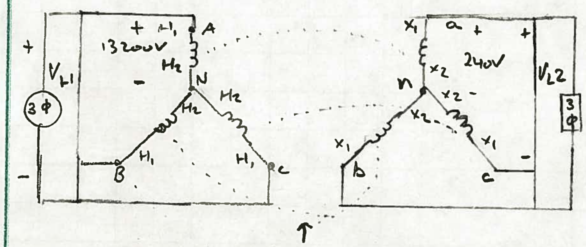
$S_T = 48kVA$

360/120 V



$S_T = 7.2kVA$

$\boxed{3\Phi \text{ TRANSFORMER}}$



$\bar{V}_{L1} = \sqrt{3}\,\bar{V}_{AN} = 22863\text{ V}$

$\bar{V}_{L2} = \sqrt{3}\,\bar{V}_{an} = 415.7\text{ V}$

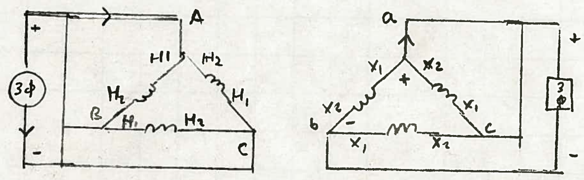
$\bar{I}_{AN} = \frac{59400}{13200} = 4.5\text{A}$

$\bar{I}_{na} = \frac{59400}{240} = 247.5\text{A}$

INTO H_1 → OUT OF X_1

3 x 13200/240V 60 Hz 59.4 kVA

$S_T = \sqrt{3}\,|\bar{V}_L|\,|\bar{I}_L| = 178.2\text{ kVA}$ (SAME FOR 1 & 2)

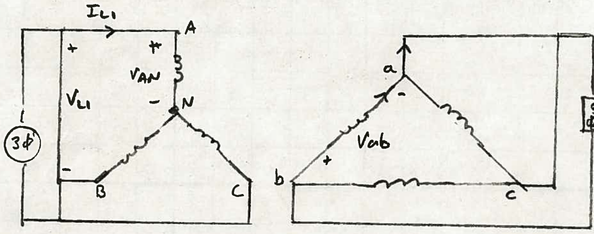


$\bar{I}_{ab} = \frac{59400}{240} = 247.5A$ $\bar{I}_{L2} = \sqrt{3}\cdot 247.5A = 428.7A$

$\bar{I}_{AB} = \frac{59400}{13200} = 4.5A$ $\bar{I}_{L1} = \sqrt{3}\cdot 4.5A = 7.79A$

$\bar{V}_{ab} = 240V$ $\bar{V}_{AB} = 13200\text{ V}$

$S_T = \sqrt{3}\,|V_L|\,|I_L| = 178.2\text{ kVA}$



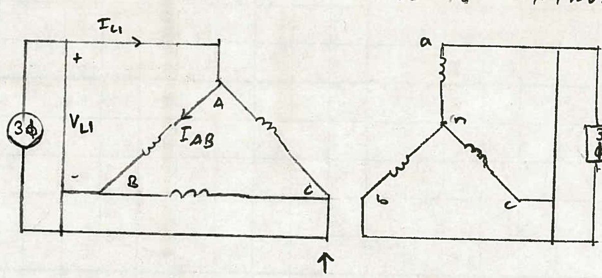
$\bar{V}_{L1} = 13200\text{ V}$ $\bar{V}_{AN} = \frac{13200}{\sqrt{3}} = 7621\text{ V}$

$\bar{I}_{L1} = \frac{59400}{13200} = 4.5A$ $\bar{V}_{ab} = 240\text{ V}$

$S_T = 3\,(13200V\cdot 4.5A) = 178.2\text{ kVA}$

3 x 7621V/240V 60 Hz 59.4 kVA

a = 31.8



$\bar{V}_{L1} = 13200V$ $\bar{V}_{AB} = 13200V$

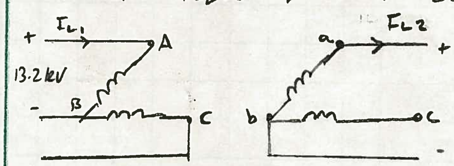
$\bar{I}_{L1} = \frac{59400}{13200} = 4.5A$

$\bar{V}_{an} = \frac{V_L}{\sqrt{3}} = 138.6\text{ V}$

$S_T = (13200V\cdot 4.5A\cdot 3) = 178.2\text{ kVA}$

3 x 13200V/138.6V 60 Hz 59.4 kVA

a = 95.2

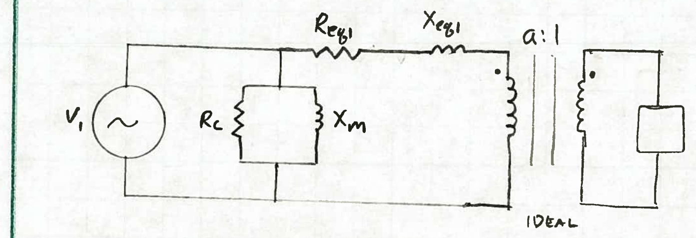


$I_{L2} = \frac{59400}{\sqrt{3}\cdot 240} = 142.9\text{A}$

$I_{L1} = \frac{59400}{\sqrt{3}\,13200} = 2.6A$

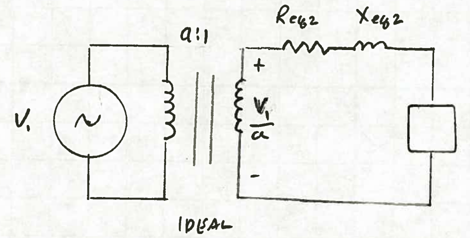
$S_T = I_{L1}\cdot V_{L1} = 34320\text{ VA} \neq 2S_\phi$

2 x 13200V/240V 60 Hz 34320 VA

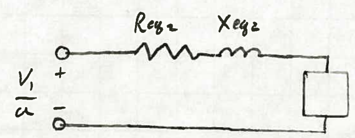


$R_{eq1} = R_1 + a^2 R_2 \qquad X_{eq1} = X_1 + a^2 X_2$

$\boxed{\text{REAL TRANSFORMER}}$

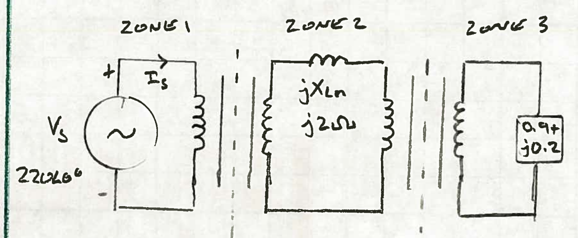


$R_{eq2} = \dfrac{R_{eq1}}{a^2} \qquad X_{eq2} = \dfrac{X_{eq1}}{a^2}$



$\dfrac{V_1}{a} = \sqrt{(V_{Ld}\cos\phi + I_{Ld}R_{eq2})^2 + (V_{Ld}\sin\phi \pm I_{Ld}X_{eq2})^2}$

$\boxed{\text{PER UNIT}}$



30 kVA
240/480V
$Z_{eq1} = j0.768\Omega$

20 kVA
460/115V
$Z_{eq1} = j1.058\Omega$

CHOOSE $\tilde{S}_b = 30\,\text{kVA}$ AND $V_{b2} = 480\,\text{V}$ IN ZONE 2

$V_{b1} = V_{b2}\dfrac{240}{480} = 240\,\text{V} \qquad V_{b3} = V_{b2}\dfrac{115}{460} = 120\,\text{V}$

$Z_{b1} = \dfrac{V_{b1}^2}{S_b} = \dfrac{240^2}{30000} = 1.92\Omega \qquad Z_{b3} = \dfrac{V_{b3}^2}{S_b} = 0.48\Omega$

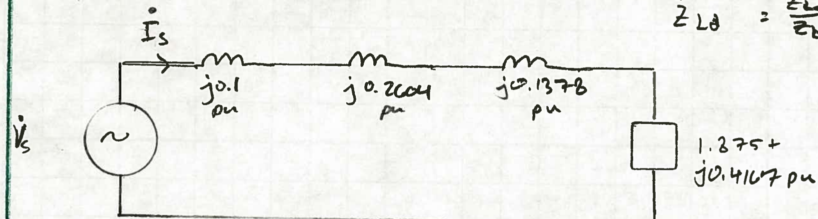
$Z_{b2} = \dfrac{V_{b2}^2}{S_b} = 7.68\Omega$

$\dot{Z}_{T1} = \dfrac{Z_{eq}(T1)}{Z_{b2}} = \dfrac{j0.768}{7.68} = j0.1\,\text{pu}$

$\dot{Z}_{Ln} = \dfrac{Z_{Ln}}{Z_{b2}} = \dfrac{j2}{7.68} = j0.2604\,\text{pu}$

$\dot{Z}_{T2} = \dfrac{Z_{eq}(T2)}{Z_{b2}} = \dfrac{j1.058}{7.68} = j0.1378\,\text{pu}$

$\dot{Z}_{Ld} = \dfrac{Z_{Ld}}{Z_{b3}} = \dfrac{0.9+j0.2}{0.48} = 1.875 + j0.4167\,\text{pu}$



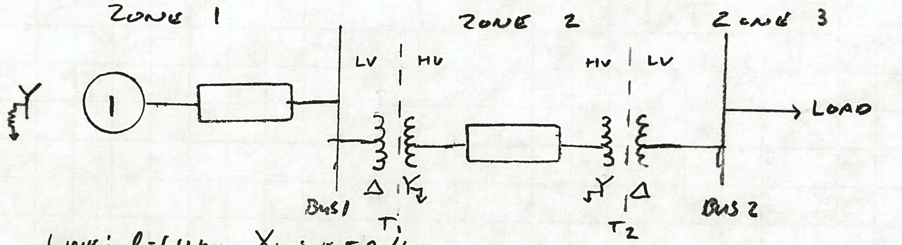
$\dot{V}_s = \dfrac{V_s}{V_{b1}} = \dfrac{220\angle 0°}{240} = 0.9167\angle 0°\,\text{pu} \qquad \dot{I}_s = \dfrac{\dot{V}_s}{\Sigma \dot{Z}} = 0.4395\angle -26.01°\,\text{pu}$

$I_{b1} = \left(\dfrac{S_b}{V_{b1}}\right)^* = \dfrac{30000}{240} = 125\,\text{A} \qquad I_{b2} = \left(\dfrac{S_b}{V_{b2}}\right)^* = \dfrac{30000}{480} = 62.5\,\text{A} \qquad I_{b3} = \left(\dfrac{S_b}{V_{b3}}\right)^* = \dfrac{30000}{120} = 250\,\text{A}$

$I_{Ld} = \dot{I}_s I_{b3} = 109.88\angle -26.01\,\text{Arms} \qquad I_{Ln} = \dot{I}_s I_{b2} = 27.45\angle -26.01°\,\text{Arms}$

$I_s = \dot{I}_s I_{b1} = 54.94\angle -26.01°\,\text{Arms}$

ONE LINE DIAGRAM



LINE: $l = 64\,km$ $X_{Ln} = 0.5\,\Omega/km$

GENERATOR: $\tilde{S} = 300\,MVA$ $V = 20\,kV$ $\dot{X}_{GEN} = 0.2$ pu

T_1: $\tilde{S} = 350\,MVA$ $230\,kV\,Y / 20\,kV\,\Delta$ $\dot{X}_{T1} = 0.1$ pu

T_2: $3 \times 1\phi$. EACH $\tilde{S}_\phi = 100\,MVA$ $127\,kV\,Y / 13.2\,kV\,\Delta$ $\dot{X}_{T2} = 0.1$ pu

LOAD: $\tilde{S} = 250\,kVA$ @ pf: 0.8 LAGGING $V = 13.2\,kV$

1. CONVERT 1ϕ UNITS TO 3ϕ BANK

T_2: $V_{AN} = \sqrt{3}\,V_{AB} = 220\,kV$ $\tilde{S}_T = 3 S_\phi = 300\,MVA$ $\dot{X}_{T2} = 0.1$ pu

T_2: $\tilde{S}_T = 300\,MVA$ $220\,kV / 13.2\,kV$ $\dot{X}_{T2} = 0.1$ pu

2. DEFINE $\tilde{S}_b$ FOR ENTIRE SYSTEM AND V_B FOR A ZONE.

LET $S_b = 500\,MVA$ AND LET $V_{b2,AB} = 230\,kV = V_{b0,HV}$ AND $20\,kV = V_{b0,LV}$

3. CALCULATE pu VALUES $V_{bi} = \frac{V_i}{V_0} \cdot V_{b0}$

$V_{b1} = V_{b0}\frac{V_1}{V_0} = 230\,kV \cdot \frac{20}{230} = 20\,kV$

$V_{b2} = V_{b0}\frac{V_2}{V_0} = 230\,kV \cdot \frac{13.2}{220} = 13.8\,kV$

CHANGE OF BASIS

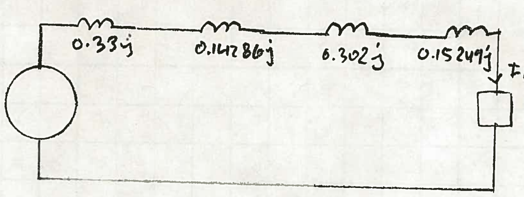
GENERATOR': $\dot{X}'_{GEN} = \dot{X}_{GEN}\left(\frac{V_b}{V_b'}\right)^2\left(\frac{\tilde{S}_b'}{\tilde{S}_{GEN}}\right) = 0.2\left(\frac{20\,kV}{20\,kV}\right)^2\left(\frac{500\,MVA}{300\,MVA}\right) = 0.33$ pu

T1: $\dot{X}'_{T1} = \dot{X}_{T1}\left(\frac{V_b}{V_b'}\right)^2\left(\frac{\tilde{S}_b'}{\tilde{S}_{GEN}}\right) = 0.1\left(\frac{20\,kV}{20\,kV}\right)^2\left(\frac{500\,MVA}{300\,MVA}\right) = 0.14286$ pu

$Z_{b2} = \frac{V_{b0}^2}{\tilde{S}_b} = \frac{230^2}{500} = 105.8$ $X_{Ln} = 64\,km \cdot \frac{0.5\,\Omega}{km} = 32\,\Omega$ $\dot{X}_{Ln} = \frac{X_{Ln}}{Z_{b2}} = 0.302$ pu

T2: $\dot{X}'_{T2} = \dot{X}_{T2}\left(\frac{V_b}{V_b'}\right)^2\left(\frac{\tilde{S}_b'}{\tilde{S}_{GEN}}\right) = 0.1\left(\frac{230\,kV}{230\,kV}\right)^2\left(\frac{500\,MVA}{300\,MVA}\right) = 0.15249$ pu

4. DRAW EQUIVALENT PER-PHASE DIAGRAM
5. FIND ACTUAL VALUES



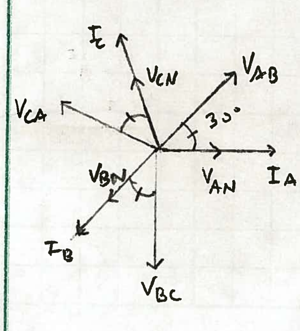
$\dot{V}_{Ld} = \frac{V_{Ld}}{V_{b3}} = \frac{13.2\angle 0°}{13.8} = 0.9565\angle 0°$ pu

$\dot{S}_{Ld} = \frac{\tilde{S}_{Ld}}{\tilde{S}_b} = \frac{250\angle 36.87°}{500} = 0.5\angle 36.87°$ pu

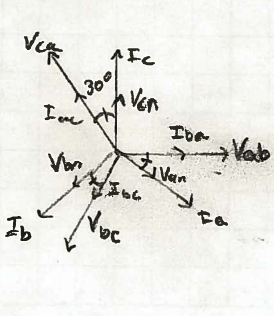
$\dot{I}_{Ld} = \left(\frac{\dot{S}_{Ld}}{\dot{V}_{Ld}}\right)^* = 0.5227\angle -36.87°$ pu

$\dot{V}$ AT GEN. TERMINALS $= \dot{V}_{Ld} + \dot{I}_{Ld}(\dot{X}'_{T2} + \dot{X}'_{Ln} + \dot{X}_{T1}) = 1.171\angle 12.327°$ pu $V_{GEN} = \dot{V}_{GEN} \cdot \dot{V}_{b1} = 23.42\angle 12.327°$ V_{rms}

6. ADD PHASE SHIFT IF NEEDED.



HV Y



LV Δ

HV: $V_{AB} = V_{AN}\sqrt{3}\angle 30°$

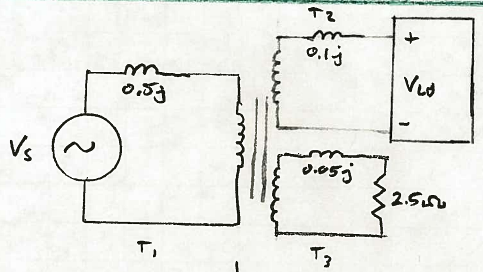
V_{AB} HV LEADS V_{ab} LV BY 30°

$\bar{I}_{Ln} = \dot{I}_{Ln} \cdot I_{b2} = 0.5227\angle -36.87° \cdot \frac{\tilde{S}_b}{\sqrt{3}\,V_{b2}} = 656\angle -36.87°$

$= 656\angle -6.87°\,A$

$\bar{I}_{Ld} = \dot{I}_{Ld} \cdot I_{b3} = 0.5227\angle -36.87° \cdot \frac{\tilde{S}_b}{\sqrt{3}\,V_{b3}} = 10934.72\angle -36.87\,A$

STAR EQUIVALENT CIRCUIT



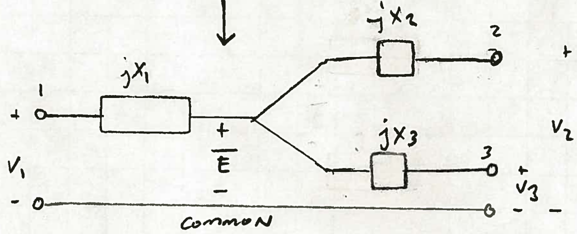
T_1: $\tilde{S} = 100\,kVA$ $V = 1000\,V$

T_2: $\tilde{S} = 50\,kVA$ $V = 500\,V$

T_3: $\tilde{S} = 50\,kVA$ $V = 250\,V$

LOAD: $\tilde{S} = 50\,kVA$ @ pf = 1.0 $\bar{V}_{RATED} = 500\angle 0°$

$$\frac{\mathcal{E}_1}{N_{E1}} = \frac{\mathcal{E}_2}{N_{E2}} = \frac{\mathcal{E}_3}{N_{E3}}$$



CHOOSE $\tilde{S}_b = 100\,kVA$ AND $V_{b1} = 1000\,V = V_{bo}$

$$V_{b2} = V_{bo} \cdot \frac{V_2}{V_1} = 500\,V \qquad V_{b3} = V_{bo} \frac{V_3}{V_1} = 250\,V$$

$$Z_{b1} = \frac{V_{b1}^2}{\tilde{S}_b} = 10\,\Omega \qquad Z_{b2} = \frac{V_{b2}^2}{\tilde{S}_b} = 2.5\,\Omega \qquad Z_{b3} = \frac{V_{b3}^2}{\tilde{S}_b} = 0.625\,\Omega$$

$$Z_{b2.5} = \frac{V_{b3}^2}{\tilde{S}_b} = 0.625\,\Omega$$

$$\dot{Z}_1 = \frac{Z_1}{Z_{b1}} = 0.05j\ pu \qquad \dot{Z}_2 = \frac{Z_2}{Z_{b2}} = 0.04j\ pu \qquad \dot{Z}_3 = \frac{Z_3}{Z_{b3}} = 0.08j\ pu \qquad \dot{Z}_{2.5} = \frac{Z_{2.5}}{Z_{b3}} = 4\ pu$$

$$\dot{\bar{V}}_{Ld} = \frac{V_{Ld}}{V_{b2}} = 1\angle 0°\ pu \qquad \dot{\bar{I}}_{Ld} = \left(\frac{\dot{\tilde{S}}_{Ld}}{\dot{\bar{V}}_{Ld}}\right)^* = 0.5\angle 0°\ pu$$

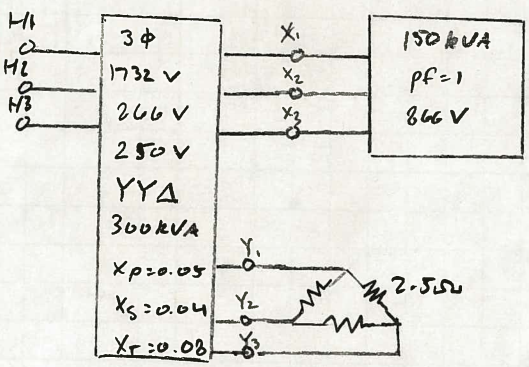
$$\dot{\bar{E}} = \dot{\bar{V}}_{Ld} + \dot{\bar{I}}_{Ld} + jX_2 = 1 + 0.02j\ pu$$

$$\dot{\bar{I}}_3 = \frac{\dot{\bar{E}}}{jX_3 + R_3} = 0.25\angle 0°\ pu$$

$$\dot{\bar{I}}_1 = \dot{\bar{I}}_2 + \dot{\bar{I}}_3 = 0.75\angle 0°\ pu$$

$$\dot{\bar{V}}_1 = \dot{\bar{E}} + \dot{\bar{I}}_1 jX_1 = 1.00165\angle 3.29°\ pu$$

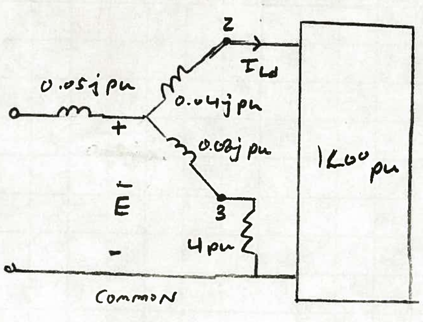
$$\bar{V}_S = \dot{\bar{V}}_1 V_{b1} = 1001.65\angle 3.29°\ V_{rms}$$



CHOOSE $\tilde{S}_b = 300\,kVA$

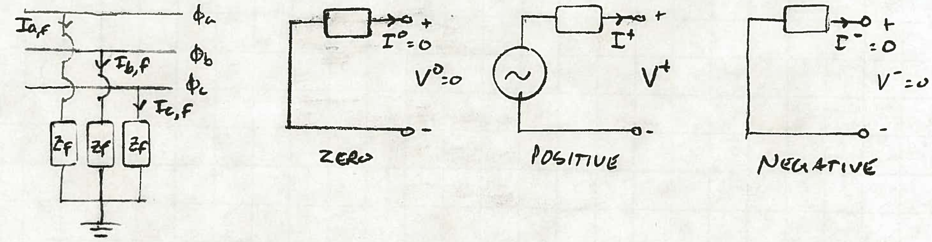
$V_{b1,AB} = 1732\,V$ $V_{b2,AB} = 866\,V$ $V_{b3,AB} = 250\,V$

$$Z_Y = \frac{Z_\Delta}{3} = \frac{2.5}{3} \qquad \dot{Z}_Y = \frac{Z_Y}{Z_b} = \frac{Z_Y}{\frac{V_{b3AB}^2}{\tilde{S}_b}} = 4\ pu$$

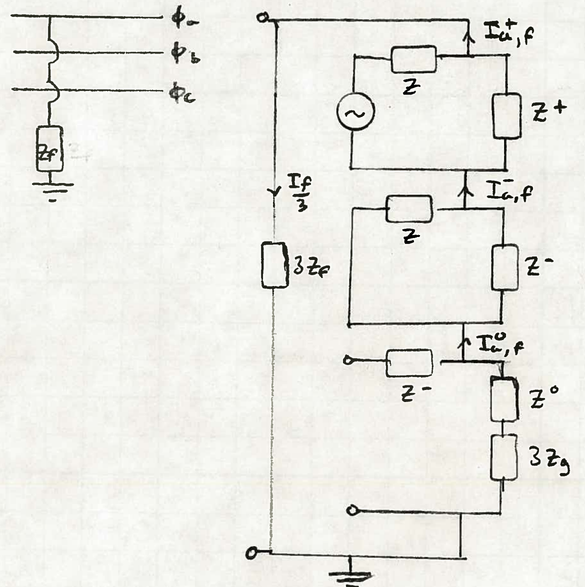


$$\dot{I}_{Ld} = \frac{\dot{\tilde{S}}_{Ld}}{\dot{\bar{V}}_{Ld}} = 0.5\angle 0°$$

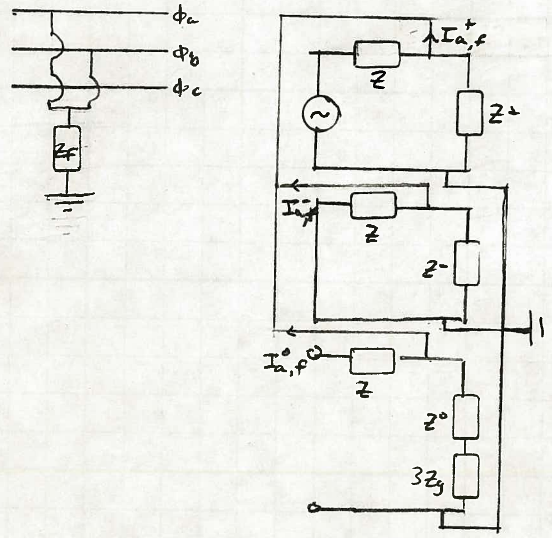
$$\dot{\bar{E}} = \dot{\bar{V}}_{Ld} + \dot{\bar{I}}_{Ld} \cdot jX_2 = 1.0002\angle 1.146°\ pu$$



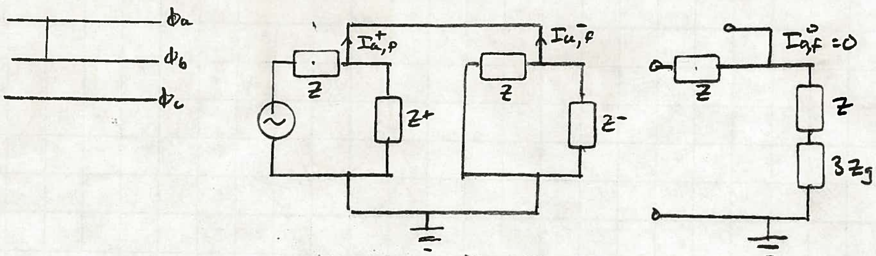
3Φ FAULT



SLG FAULT

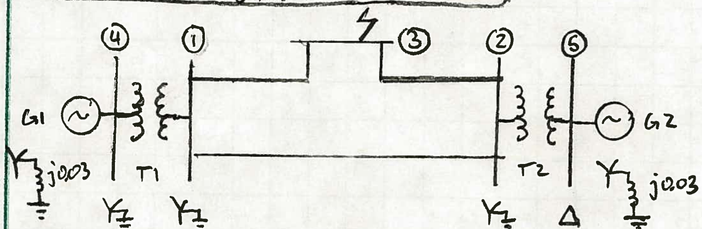


DLG FAULT

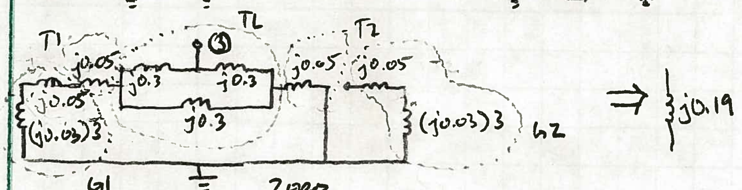


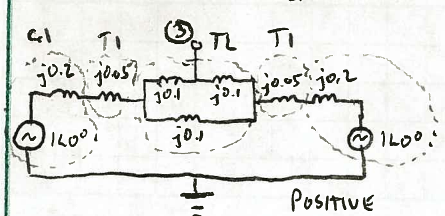
L-L FAULT

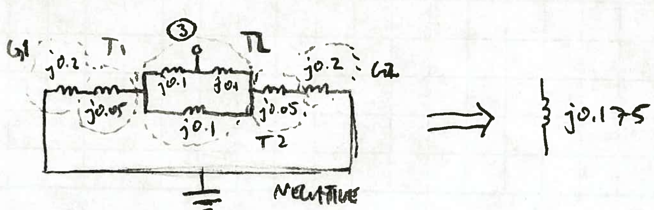
ONE LINE DIAGRAM EXAMPLE

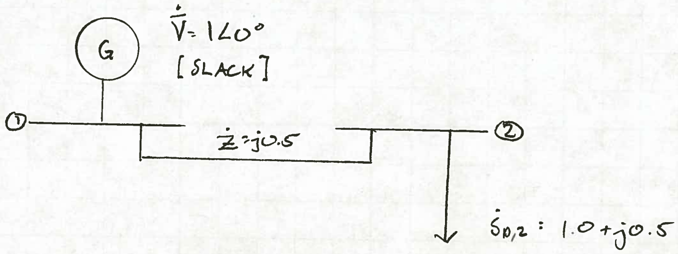


ITEM	MVA	kV	X^+	X^-	X^0
G1	100	25	0.2	0.2	0.05
G2	100	13.8	0.2	0.2	0.05
T1	100	25/230	0.05	0.05	0.05
T2	100	13.8/230	0.05	0.05	0.05
TL12	100	230	0.1	0.1	0.3
TL13	100	230	0.1	0.1	0.3
TL23	100	230	0.1	0.1	0.3









POWER FLOW ANALYSIS

FIND $|V_2^{(1)}|$, $\phi_2^{(1)}$, $\tilde{P}_{12}^{(1)}$

$$\dot{Y}_{Ln} = (\dot{Z}_{Ln})^{-1} = -j2 \qquad Y_{Bus} = j\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

GAUSS - SEIDEL: $\quad V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\left(\frac{S_i}{V_i^{(k)}}\right)^* - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j^{(k)}\right]$

Assume flat profile: $|V_2^{(0)}| = 1 \quad \phi_2^{(0)} = 0° \qquad Y_{ii} = Y_{22} = -j2 \qquad S_i = S_{D,2} = -(1.0 + j0.5)$

$$\dot{V}_2^{(1)} = \frac{1}{-j2}\left[\left(\frac{-1-j0.5}{1.0\angle 0°}\right)^* - j2(1\angle 0°)\right] = 0.901\angle -33.7°$$

$$\dot{P}_{12} = V_1(Y_{11}^* V_1^* + Y_{12}^* V_2^*) = 1.12\angle 26.57°$$

NEWTON - RAPHSON $\quad \tilde{P}_i = |V_i| \sum_{j=1}^{n} |V_j| jB_{i,j} \sin\phi_{ij} \qquad \tilde{Q}_i = |V_i| \sum_{j=1}^{n} |V_j| \cdot jB_{i,j} \cos\phi_{i,j} - |V_i|^2 B_{i,i}$

$$B = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} \phi_2 \\ |V_2| \end{bmatrix} \qquad \vec{f}(\vec{x}) = \begin{bmatrix} P_2(\vec{x}) \\ Q_2(\vec{x}) \end{bmatrix} \qquad \text{FLAT PROFILE: } \vec{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\tilde{P}_2(\vec{x}) = |V_2||V_1| B_{21} \sin(\phi_2 - \phi_1) = 2|V_2| \sin\phi_2$$

$$\tilde{Q}_2(\vec{x}) = -\left[|V_2||V_1| B_{21} \cos(\phi_2 - \phi_1) + |V_2||V_2| B_{22}\right] = -2|V_2|\cos\phi_2 + 2|V_2|^2$$

$$J(\vec{x}) = \begin{bmatrix} \frac{\partial P_2}{\partial \phi_2} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \phi_2} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} = \begin{bmatrix} -2|V_2|\cos\phi_2 & 0 \\ 0 & -2|V_2|\cos\phi_2 + 2|V_2|^2 \end{bmatrix}$$

$$J(\vec{x}^{(0)}) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \qquad \vec{f}(\vec{x}^{(0)}) = \begin{bmatrix} -1.0 \\ -0.5 \end{bmatrix} \qquad \vec{x}^{(k+1)} = x^{(k)} - J(\vec{x}^{(k)})^{-1} \vec{f}(\vec{x}^{(k)})$$

$$\vec{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} -1.0 \\ -0.5 \end{bmatrix} = \begin{bmatrix} -28.650° \\ 0.75 \end{bmatrix} = \begin{bmatrix} \phi_2^{(1)} \\ |V_2|^{(1)} \end{bmatrix} \qquad \dot{P}_{12}^{(1)} = 0.99\angle 43.55°$$

FAST DECOUPLE

$$J(\vec{x}) = \begin{bmatrix} J1(\vec{x}) & 0 \\ 0 & J2(\vec{x}) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

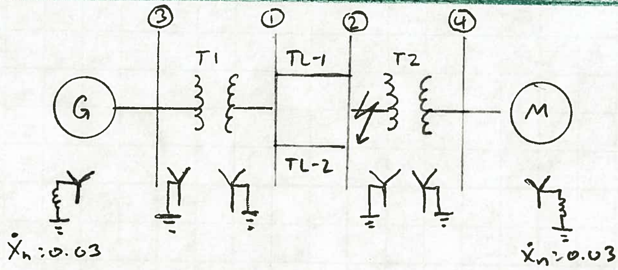
$$\phi^{(k+1)} = \phi^{(k)} - J1(\vec{x}^{(k)})^{-1} \frac{\Delta\tilde{P}^{(k)}}{|V|} \qquad |V|^{(k+1)} = |V|^{(k)} - J4(\vec{x}^{(k)})^{-1} \frac{\Delta\tilde{Q}^{(k)}}{|V|}$$

$\Delta\tilde{P}^{(0)} = [-1] \qquad \phi^{(0)} = [0] \qquad |V|^{(0)} = [1] \qquad \Delta\tilde{Q}^{(0)} = [-0.5]$

$\phi^{(1)} = -28.65° \qquad |V|^{(1)} = 0.75$

$\dot{\tilde{P}}_{12} = 0.99\angle 43.55°$

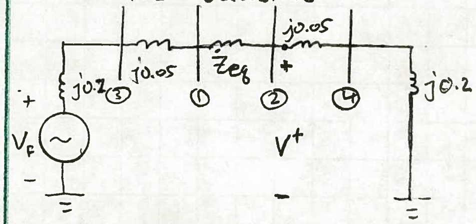
FAULT ANALYSIS



	MVA	KV	$\dot{X}^+$	$\dot{X}^-$	$\dot{X}^0$
G	100	25	0.2	0.2	0.05
M	100	13.8	0.2	0.2	0.05
T1	100	25/230	0.05	0.05	0.05
T2	100	13.8/230	0.05	0.05	0.05
TL-1	100	230	0.1	0.1	0.3
TL-2	100	230	0.2	0.2	0.3

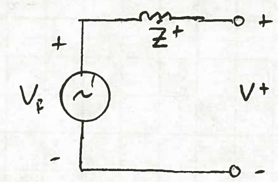
$\dot{V}_F = 1.05\angle 0°$

POSITIVE SEQUENCE



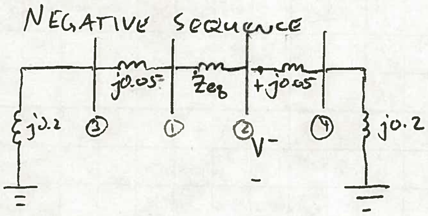
$\dot{Z}_{eq} = j0.1 \| j0.2 = j0.067$

THEVENIN EQUIVALENT



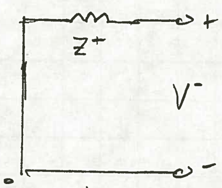
$\dot{Z}^+ = (j0.067 + j0.25) \| j0.25 = j0.1397$

NEGATIVE SEQUENCE



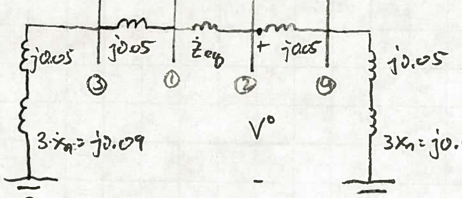
$\dot{Z}_{eq} = j0.1 \| j0.2 = j0.067$

THEVENIN EQUIVALENT



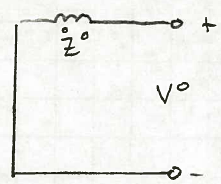
$\dot{Z}^- = (j0.067 + j0.25) \| j0.25 = j0.1397$

ZERO SEQUENCE



$\dot{Z}_{eq} = j0.3 \| j0.3 = j0.15$

THEVENIN EQUIVALENT



$\dot{Z}^0 = (j0.09 + j0.05 + j0.05 + j0.15) \| (j0.05 + j0.05 + j0.09)$
$= j0.122$

3ϕ SC BALANCED

$V^0 = V^- = 0 \quad I^0 = I^- = 0$

$\dot{I}^+ = \frac{\dot{V}_F}{\dot{Z}^+} = \frac{1.05\angle 0°}{j0.1397} = 7.52\angle -90°$

$\dot{I}^{0+-} = \langle 0, 7.52\angle -90°, 0 \rangle$

$\dot{I}^f_{abc} = A \cdot I^{0+-} = \langle 7.52\angle 90°, 7.52\angle 150°, 7.52\angle 30° \rangle$

SLG

$I^0 = I^+ = I^- = \frac{\dot{V}^+}{\dot{Z}^+ + \dot{Z}^- + \dot{Z}^0} = \frac{1.05\angle 0°}{j0.4013} = 2.617\angle -90°$

$\dot{I}^{0+-} = \langle 2.617\angle -90°, 2.617\angle -90°, 2.617\angle -90° \rangle$

$\dot{I}^f_{abc} = A \cdot \dot{I}^{0+-} = \langle 7.85\angle -90°, 0, 0 \rangle$

LL

$I^0 = 0 \quad I^+ = -I^- = \frac{V^+}{Z^+ + Z^-} = \frac{1.05\angle 0°}{j0.2794} = 3.758\angle -90°$

$\dot{I}^{0+-} = \langle 0, 3.758\angle -90°, 3.758\angle -90° \rangle$

$\dot{I}^f_{abc} = \langle 0, -6.5, 6.5 \rangle$

DLG

$I^+ = \frac{V^+}{Z^+ + (Z^0 \| Z^-)} = \frac{1.05\angle 0°}{j0.205} = 5.127\angle 90°$

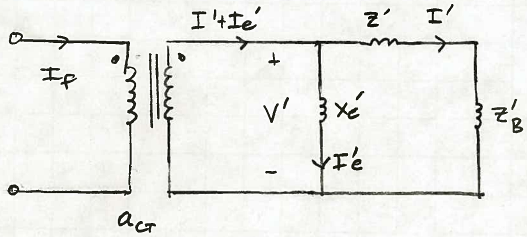
$I^- = -I^+ \left(\frac{Z^0}{Z^0 + Z^+} \right) = -5.127\angle -90° \,(0.466) = 2.389\angle 90°$

$I^0 = -(I^+ + I^-) = 2.738\angle -90°$

$\dot{I}^{0+-} = \langle 2.738\angle -90°, 5.127\angle 90°, 2.389\angle 90° \rangle$

$\dot{I}^f_{abc} = \langle 0, 7.696\angle 147.75°, 7.696\angle 32.25° \rangle$

CURRENT TRANSFORMER

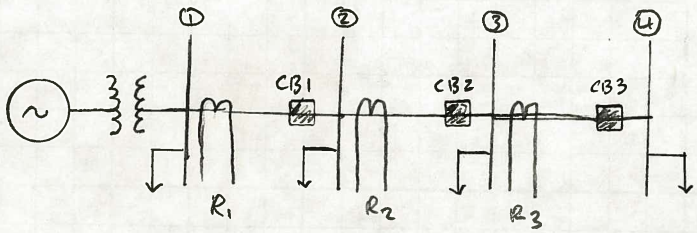


I' = SECONDARY OUTPUT
Z' = SECONDARY LEAK IMPEDANCE
Z_B : IMPEDANCE OF TERM. DEVICE

1. $a_{CT} \rightarrow Z'$ (FIG. 10.8)
2. ASSUME I', Z_B'
3. $V' = (Z' + Z_B')I'$
4. $(V', a_{CT}) \rightarrow I_e'$ (FIG 10.8)
5. $I_f = a_{CT}(I' + I_e')$
6. $e = \frac{I'_e}{I_e' + I'}$

$(I' + I_e') = \frac{I_{RATED}}{a} \approx I'$

OVERCURRENT RELAY PROTECTION



$\frac{I_f}{a_{CT}} = (I' + I_e')_f$

$M = \frac{(I' + I_e')_f}{T}$

$t_{clear} = t_{trip} + t_{cyc}$

$t_{cyc} = \frac{N_{cyc}}{60}$

$t = TD(0.0963 + \frac{3.88}{M^2 - 1})$

$R_1 : a_{CT} : 800:5 \quad I_R = 700A \qquad R_2 : a_{CT} : 600:5 \quad I_R = 350A \qquad R_3 : a_{CT} : 600:5 \quad I_R = 250A$

EACH CB HAS $N_{cyc} = 5 \rightarrow t_{cyc} = \frac{5}{60}$ s $= 0.083$ s
$t_{coord} = 0.3$ s

$CT_3 : (I' + I_e') = \frac{I_R}{a_{CT}} = \frac{250}{600/5} = 2.08 \approx I' \Rightarrow T_3 = 2.5 \qquad CT_2 : \frac{I_R}{a_{CT}} = \frac{350}{600/5} = 2.91 \Rightarrow T_2 = 3$

$CT_1 : \frac{I_A}{a_{CT}} : \frac{700}{800/5} = 4.38 \Rightarrow T_1 = 5$

FAULT SCENARIO: FAULT IN 3-4, $I_f = 2600$ A, $TD_3 = 0.5$

$(I' + I'_e)_f : \frac{2600A}{600/5} = 21.67A \qquad M_3 : \frac{(I' + I'_e)_f}{T_3} = \frac{21.67}{2.5} = 8.67$

$(M_3 = 8.67, TD_3 = 0.5) \longrightarrow t_{trip} = 0.1$ s (FIG 10.12)

$t_{clear} = t_{trip} + t_{cyc} = 0.1s + 0.083s = 0.183s$

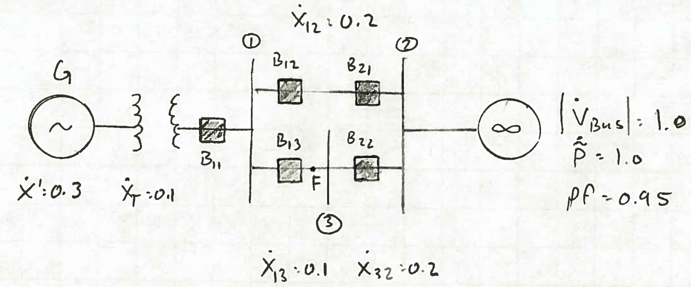
FAULT SCENARIO: FAULT IN 3-4, $I_f = 2600$ A, $TD_3 = 0.5$, CB_3 MALFUNCTIONS

$t'_{trip} = t_{clear} + t_{coord} = 0.183s + 0.3s = 0.483s$

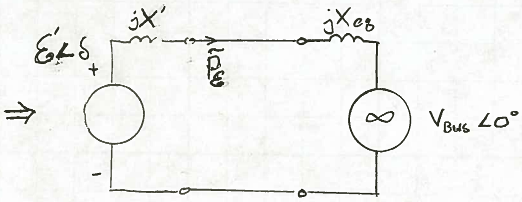
$(I' + I'_e)_f = \frac{I_f}{a_{CT}} = \frac{2600A}{600/5} = 21.67A$

$M_2 = \frac{(I' + I'_e)_f}{T_2} = \frac{21.67}{3} = 7.22$

$(M_2 = 7.22, t'_{trip} = 0.483s) \longrightarrow TD_2 = 2$ (FIG 10.12)



$\Rightarrow$



PRE-FAULT:

$\dot{X}_{eq} = (0.3 + 0.1 + 0.2) \| (0.1 + 0.2) = 0.52$

$I = \frac{|\tilde{P}_m|}{|V_{Bus}| \cdot pf} \angle -\cos^{-1}(pf) = \frac{1.0}{1.0 \cdot 0.95} \angle -\cos^{-1}(0.95) = 1.053 \angle -18.2°$ pu

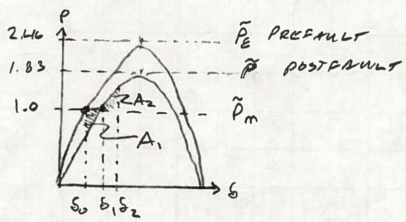
$\mathcal{E}' = V_{Bus} + jX_{eq} I = 1.0 \angle 0° + j0.52 (1.053 \angle -18.2°) = 1.28 \angle 23.9°$ pu

$\tilde{P}_E = \frac{|\mathcal{E}'||V_{Bus}|}{X_{eq}} \sin\delta = \frac{1.28(1.0)}{0.52} \sin\delta = 2.46 \sin\delta$ pu

FAULT SCENARIO: B_{12} INADVERTENTLY OPENS

$\dot{X}_{eq} = 0.3 + 0.1 + 0.1 + 0.2 = 0.7$

$\tilde{P} = \frac{1.28(1.0)}{0.7} \sin\delta = 1.83 \sin\delta$



$A_2 = A_1$

$2.46 \sin\delta_0 = 1 \longrightarrow \delta_0 = 23.95°$

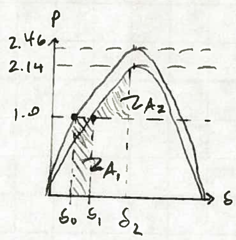
$1.83 \sin\delta_1 = 1 \longrightarrow \delta_1 = 33.12°$

$\int_{23.95°}^{33.12°} (1 - 1.83 \sin\delta)\, d\delta = \int_{33.12°}^{\delta_2} (1.83 \sin\delta - 1)\, d\delta \longrightarrow \delta_2 = 42.63°$

FAULT SCENARIO: 3ϕ TO GROUND FAULT AT F, B_{13} AND B_{12} OPEN

$\dot{X}_{eq} = 0.3 + 0.1 + 0.2 = 0.6$

$\tilde{P} = \frac{1.28(1.0)}{0.6} \sin\delta = 2.14 \sin\delta$



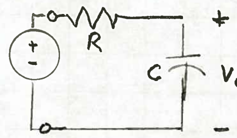
$2.46 \sin\delta_0 = 1 \longrightarrow \delta_0 = 23.95°$

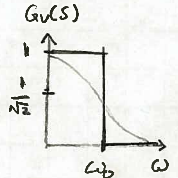
$2.14 \sin\delta_1 = 1 \longrightarrow \delta_1 = 27.90°$

$\int_{23.95°}^{27.90°} 1\, d\delta = \int_{27.90°}^{\delta_2} (2.14 \sin\delta)\, d\delta \longrightarrow \delta_2 = 43.33°$

VARIABLE FREQ. RESPONSE

LOWPASS



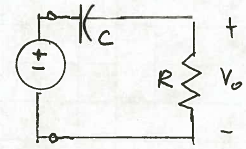


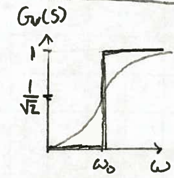
$G_V(s) = \frac{1}{1+j\omega\tau}$ $\tau = RC$

$\phi(\omega) = \tan^{-1}(\omega\tau)$

$|G_V(s)| = \frac{1}{\sqrt{Re^2 + Im^2}} = \frac{1}{\sqrt{1+(\omega\tau)^2}}$

HIGHPASS



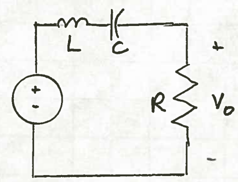


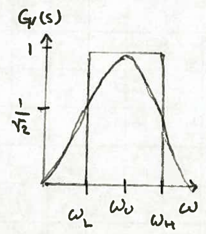
$G_V(s) = \frac{j\omega\tau}{1+j\omega\tau}$ $\tau = RC$

$\phi(\omega) = \frac{\pi}{2} - \tan^{-1}(\omega\tau)$

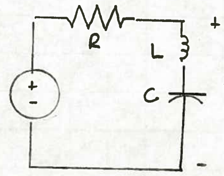
$|G_V(s)| = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$

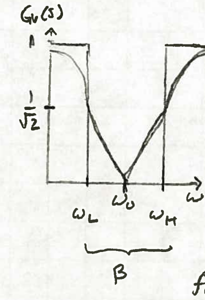
BAND PASS





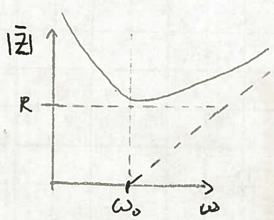
$G_V(s) = \frac{R}{R + j(\omega L - \frac{1}{\omega C})}$



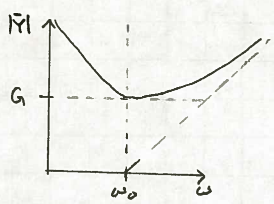


$f_Q \uparrow \Rightarrow \beta \downarrow$

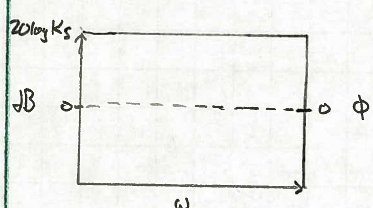
RESONANCE



SERIES

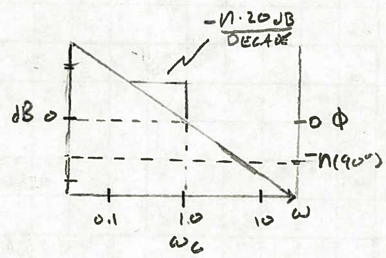


PARALLEL



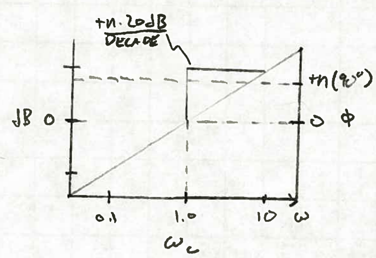
$H(j\omega) = K_s$

FREQ. INDEPENDENT



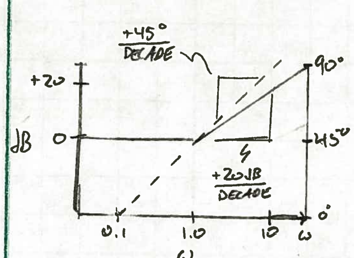
$H(j\omega) = \frac{1}{(\tau j\omega)^n}$ $\omega_c = \frac{1}{\tau}$

POLE AT ORIGIN



$H(j\omega) = \frac{(\tau j\omega)^n}{1}$

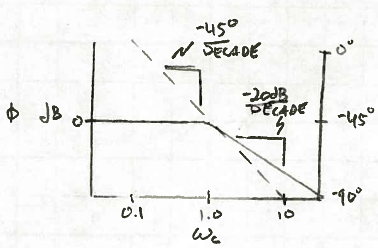
ZERO AT ORIGIN



$dB = 20\log_{10}|1 + j\omega\tau|$

$\phi = \tan^{-1}(\omega\tau)$

SIMPLE ZERO



$dB = 20\log_{10}|(1 + j\omega\tau)^{-1}|$

$\phi = \tan^{-1}(\omega\tau)$

SIMPLE POLE